**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer :

Mean = 45 mins

X = 60 mins – 10 mins = 50 mins

Std = 8 mins

Z(can meet) = z(x<50) = (50-45)/8 = 5/8 = 0.625

P(z=0625) = 0.734

P(cannot meet) = 1 – 0.734 = 0.266

**Option B**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**FALSE**, since the data is normally distributed most of the value lie between 38 and 44(mean+std=38+6). Beyond 44 the data count will be less.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Emp% under age 30 = 50% - 34.13% = 15.87%

Count below 30 = 0.1587 \* 400 = 63

TRUE, the training program can attract atleast 36 employees below 30

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Both X1 and X2 are normally distributed with same mean and same standard deviation. But multiplying same factor will not hold any influence on Y value and R value can be lesser than expected. But in case of X1 + X2, both might have 2 different datasets and values, therefore adding them both as a factor for Y, R value might improve, unless X1 and X2 are perfectly co-related.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Mean = 100

Std Dev = 20

Z = (x - 100)/20

x- 100 = 20z

x = 20z+100

* 1. excluded, resulting in 0.005 exclusion on both the end.

Z(1-0.005) = Z(0.0995) = stats.norm.ppf(0.995) = 2.5758

a = - (20 \* 2.5758) + 100

a = 100 – 51.516 = 48.484 = 48.5

b = (20 \* 2.5758) + 100

b = 100 + 51.516 = 151.516 = 151.5

Therefore (a,b) = (48.5,151.5)

**ANSWER = Option D**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Mean profit = (7+5)\*45 = 12 \*45 = 540 million rupees

Std Dev = sqrt(9+16) \*45 = sqrt(25) \* 45 = 5 \* 45 = 225 million rupees

stats.norm.interval(0.95,540,225) = (99.00810347848784, 980.9918965215122)

Rupee range lies between 99 million to 981 million

1. Specify the 5th percentile of profit (in Rupees) for the company

5th percentile = z(-0.90) = -1.645

X= 540 + (-1.645) \* 225

X = 540 – 370.125

X = 169.87 = 170

5th percentile profit is 170 million rupees.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Loss = P(x<0)

Division 1 loss probability = stats.norm.cdf(0,5,3) = 0.04777

Division 2 loss probability = stats.norm.cdf(0,7,4) = 0.04005

Therefore Division 1 has more probability of loss